PROGRAM, TITLES AND ABSTRACT – CONFERENCE AGAY LES ROCHES ROUGES

	Monday	Tuesday	Wednesday	Thursday	Friday
9 : 30	Bernard	Kravchuk	Santachiarra	Viklund	Chevillard
10 : 30	Café	Café	Café	Café	Café
11 : 00	Cerclé	Mazac	Jakobsen	Wang	He
12 : 00	Déjeuner	Déjeuner	Déjeuner	Déjeuner	Déjeuner
16 : 30	Junnila	Vargas	Free	Lupu	Chernyak
17 : 30	Café	Café	Free	Café	Café
18 : 00	Jego	Baverez	Free	Rembado	

Guillaume Baverez and Vincent Vargas: Liouville theory: from Segal axioms to conformal blocks.

Abstract: In his "Definition of conformal field theory (CFT)", Segal introduced a set of axioms that should be satisfied by any CFT. A key feature of this mathematical structure is that the gluing of surfaces should be represented as composition of operators acting on the Hilbert space of the theory, allowing one to define the theory by induction on the topology of the surface. In the first talk, we will introduce the probabilistic formulation of Liouville CFT and show that it satisfies Segal's axioms. This formulation allows one to write the correlation functions in terms of conformal bocks, which are universal analytic functions defined on the Teichmüller space of the surface, with strong representation theoretic content. In the second talk, we will introduce the semigroup of annuli and explain its quantisation in Liouville theory. This gives a geometric and probabilistic interpretation of the action of the Virasoro algebra on the Hilbert space. We use this as input to study the conformal blocks. Based on joint works with C. Guillarmou, A. Kupiainen and R. Rhodes.

Denis Bernard: SAW, Restriction and CFT

Abstract: We shall discuss the possibility to reconstruct or constrain the conformal field theory (CFT) for self-avoiding walks (SAW), in 2D and possibly 3D, using its restriction property combined with conformal invariance and bootstrap techniques

Baptiste Cerclé: Three-point correlation functions in the $\mathfrak{sl}_3$ Toda theory

Abstract: Toda Conformal Field Theories (CFTs) form a family of 2d CFTs indexed by semisimple and complex Lie algebras. They are natural generalizations of Liouville CFT that enjoy an enhanced level of symmetry encoded by W-algebras. These theories can be rigorously defined using a probabilistic framework, which involves correlated Gaussian Multiplicative Chaos measures. In this talk I wish to explain how certain three-point correlation functions, which generalize the celebrated DOZZ formula, can be computed within this probabilistic framework for the $\mathfrak{sl}_3$ Toda Conformal Field Theory and coincide with predictions from the physics literature by Fateev-Litvinov. Along the derivation of this formula I will highlight how a new Brownian path decomposition in Euclidean spaces (generalizing the one-dimensional result of Williams) allows to describe the joint tail expansion of correlated Gaussian Multiplicative Chaos measures and extend the range of values for which the probabilistic definition of the DOZZ formula makes sense.

Dmitry Chernyak: Non-compact boundary conditions for the XXZ spin chain and conformal boundary loop models

Using the quantum group symmetry of the open XXZ spin chain, we introduce new boundary conditions by coupling the bulk Hamiltonian to an infinite-dimensional Verma module on one or both boundaries. We show that for generic values of the parameters the new boundary coupling provides a faithful representation of the blob algebra which is Schur-Weyl dual to the quantum group symmetry. Modifying the boundary conditions on both the left and the right, we obtain a representation of the (universal) two-boundary Temperley-Lieb algebra. These representations then naturally define various boundary loop models on the lattice. As an example, we specialise the deformation parameter to q=i where the model can be explicitly solved by free fermions and compute its conformal scaling limit. If time permits, we will explain how to solve this model and obtain its conformal scaling limit for arbitrary q using Bethe ansatz. Based on joint work with A. Gainutdinov, J.Jacobsen and H. Saleur (2207.12772 + unpublished work).

Laurent Chevillard: Multiplicative Chaos, and the modeling of Fluid Turbulence.

Abstract: I will review and recall the early introduction of the Gaussian Multiplicative Chaos to model the fluctuating nature of the velocity field of a turbulent flow. If some time is left, i will present recent results concerning the definition of such rough random fields as an invariant measure of some simple stochastic partial differential equations. The last part is joint work with G. Apolinario and J.-C. Mourrat.

Jesper Jacobsen: Geometrical web models

Abstract: We introduce a family of geometrical lattice models generalising the well-known loop model on the hexagonal lattice. These models have a $U_q(sl_n)$ quantum group symmetry, the loop model being the $n = 2$ case. The general models give rise to branching webs and describe, at a special point, the interfaces in Z_n symmetric spin models. We mainly discuss the $n = 3$ case of bipartite cubic webs, which is based on the Kuperberg A_2 spider. We exhibit a local vertex-model reformulation, analogous to the well-known correspondence between the loop model and the nineteen-vertex model. The local formulation allows us in particular to study the model by means of transfer matrices and conformal field theory. We find that it has a rich phase diagram, including a dense and a dilute phase that generalise those known for the loop model. Based on joint work with Augustin Lafay and Azat Gainutdinov (arXiv:2101.00282 and 2107.10106).

Antoine Jego: Multiplicative chaos of the Brownian loop soup

Abstract: The Brownian loop soup is a random collection of infinitely many Brownian- type loops in a given domain of the plane which is related to many random conformal objects. In this talk, we will be interested in thick points, that is points that have been visited usually often by the loop soup. We will first explain how one can sample a typical thick point. We will then describe the structure of the loop soup viewed from such a point. We will in particular answer the following question: is a thick point thick because of finitely many very thick loops visiting the point, or because of infinitely many loops with typical local time? Based on a joint work with E. Aïdekon, N. Berestycki and T. Lupu.

Janne Junnila: Imaginary chaos and Malliavin calculus

Abstract: I will begin by giving a short introduction to a class of random distributions known as Imaginary Gaussian Multiplicative Chaos and discussing their basic properties. The second half of the talk is more specific, and focuses on the existence of probability densities for the laws of random variables obtained by testing the chaos against a test function. The talk is based on joint work with Juhan Aru and Antoine Jego.

Petr Kravchuk: Conformal Bootstrap in two acts

Abstract: In the first half of the talk, I will give a brief introduction to the conformal bootstrap. I will discuss the basic ideas as well as review some recent results from both numerical and analytical approaches. This part of the talk will focus on the traditional approach based on reflection positivity. In the second half of the talk, I will discuss another approach, now based on positivity of the path integral measure. In particular, this will set up the formalism for the talk by Dalimil Mazac on spectra of hyperbolic manifolds.

Titus Lupu: Exact probabilities for some topological events for metric graph GFF.

Abstract: The metric graph GFF is obtained by interpolating the discrete GFF by Brownian bridges inside the edges. The metric graph GFF satisfies more exact solvability properties than the discrete GFF. In particular, I will present formulas for probabilities for some topological events on the sign clusters of the GFF on metric graphs. More precisely this is related to the $\mathbb{Z}/2\mathbb{Z}$ homology of these sign clusters. These probabilities are square roots of ratios of two determinants of Laplacians, one being the usual graph Laplacian, and the other one being a twisted Laplacian. As an example, one gets on planar annular-shapped (i.e. two-connected) graphs the probability of existence of a non-contractible sign cluster that separates the inner and the outer boundary. In the scaling limit, this particular probability agrees with the one that can be computed through the level lines of the 2D continuum GFF.

Dalimil Mazac: Geometric applications of the conformal bootstrap.

Abstract: I will explain how ideas familiar from the conformal bootstrap lead to new rigorous upper bounds on the spectral gap of the Laplacian on hyperbolic orbifolds. The bounds follow from a combination of representation theory and linear programming. In two dimensions, the bounds allow us to determine the set of spectral gaps attained by all hyperbolic orbifolds. I will also discuss the question of sharpness of linear programming bounds appearing in the conformal bootstrap. In some cases, sharpness can be proven rigorously. The method of proof is essentially identical to that used by Viazovska to solve the sphere packing problem in dimension 8. Remarkably, the method was developed by physicists independently of Viazovska almost simultaneously.

Gabriele Rembado: The sheaf of irregular covacua in the WZNW model.

Abstract: Sheaves of (co)vacua provide a formalisation of conformal blocks in the Wess–Zumino–Novikov–Witten (WZNW) model for 2d CFT. They are defined on base spaces parametrising pointed Riemann surfaces (of any genus), and were introduced by Tsuchiya–Ueno–Yamada (TUY). With the correct setup (co)vacua actually have the structure of a vector bundle equipped with a (projectively) flat connection, and the correlation functions of the model become horizontal sections for this TUY/WZNW connection. The genus-zero case of this construction, viz. the flat Knizhnik–Zamolodchikov (KZ) connection, has been particularly studied; and it found important applications beyond theoretical/mathematical Physics: most notably perhaps Drinfeld's proof of Kohno's conjecture, relating the monodromy of KZ to the braiding of the representation category of the Jimbo–Drinfeld quantum group (partly leading to Drinfeld's Fields medal).

In this talk we will briefly review this story, and introduce a smooth generalisation of Verma modules for affine Lie algebras (generalising the 'confluent' Verma modules of Jimbo–Nagoya–Sun). They provide a partial generalisation of sheaves of (co)vacua, allow for the introduction of irregular states à la Gaiotto–Teschner, and in genus zero also assemble into a flat vector bundle equipped with an 'irregular/confluent' KZ connection. If possible we will sketch relations with the deformation quantisation of nonautonomous Hamiltonian systems controlling isomonodromic deformations of meromorphic connections with wild/irregular singularities on the Riemann sphere, and with the geometric quantisation of the moduli space of such connections.

Raoul Santachiara: Liouville path integral for $c \leq 1$: insights from a toy model.

Abstract: The problem of the continuation of the Liouville path integral to values of the central charge $c \leq 1$ remains an open problem, in particular, its connection to some recently discovered bootstrap solutions (some of them remarkably applied to geometric phase transitions). In this talk I will review the approach suggested by Harlow et al. (2011), based on the Lefschetz-Pham thimble theory. We applied this approach to a toy-model. I will present some results of this analysis and I will discuss some interesting insights on the Liouville path integral continuation problem.

Fredrik Viklund: Wilson loops in a 4D lattice Higgs model

Abstract: Lattice gauge theories were first considered in the early 1970s as regularized (and rigorously defined) lattice approximations of continuum quantum field theories known as Yang-Mills theories, the latter which are still to this day lacking rigorous constructions in most physically relevant settings. In the last few years there has been a renewed interest in the rigorous analysis of lattice gauge theories in the probability community. I will discuss some background in this area, including Wilson's pure gauge theory as well as the lattice Higgs model, and then report on recent work on the behavior of Wilson loop expectations in a certain limit for these models. Based on joint works with Malin Forsström (KTH) and Jonatan Lenells (KTH).

Yilin Wang: Holography and the gradient flow of the Loewner energy.

Abstract: As a starting point of AdS/CFT correspondence, the link between the hyperbolic geometry of 3-manifolds and the conformal metrics on its boundary has been explored extensively. One basic fact is that Mobius transformations on the Riemann sphere extend to isometries of the hyperbolic 3-space $\mathbb{H}^3$ and they are in one-to-one correspondence. The Loewner energy is a Mobius invariant quantity that measures the roundness of Jordan curves. It arises from large deviation deviations of SLE0+ and is a Kahler potential on the Weil-Petersson Teichmuller space. We show that the Loewner energy equals the renormalized volume of a submanifold of $\mathbb{H}^3$ constructed using the Epstein surfaces associated with the hyperbolic metric on both sides of the curve. We also study the gradient flow of the Loewner energy. This is a work in progress with Martin Bridgeman (Boston College), Ken Bromberg (Utah), and Franco Vargas-Pallete (Yale).